Development and Control of a Double-inverted Pendulum

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***Abstract*—Control of the unstable inverted pendulum is a common research topic in the area of Dynamics and Controls. The project goal was to design, implement and test all of the mechanical, electrical, and software systems necessary to stabilize and control a double inverted pendulum. Mechanical design was completed in Solidworks and the manufactured parts were 3D printed. Electrical signal conditioning was accomplished through passive and active componentry and then integrated to a dsPIC microcontroller. All final software was developed in MATLAB Simulink then converted to C code to be used in the dsPIC. The MATLAB development environment provides an easy to use system for future attempts at controlling the system. Swing up and control of a single inverted pendulum was successful though further controller tuning could yield a better system performance. The hardware and software for a double inverted pendulum was successfully implemented though stable control was never achieved. Further tuning of the LQR controller parameters would yield a stable system.**

I. Introduction

The inverted pendulum has long been a challenging problem for researchers in the field of Control and Dynamics. It has been used as a testbed for the implementation of various control algorithms. In this project, a triple inverted pendulum on a cart will be designed and developed to be used as a test bed for control courses and Labs at the Department of Mechanical Engineering, University of Utah. The stabilization problem of the pendulum will be discussed and a control method will be implemented to stabilize the pendulum.

A. Project goals and requirements

The priority for this project was to develop a double inverted pendulum on a carriage to be used as a testbed in laboratories for control courses at University of Utah. In order to achieve this goal, four intermediate goals were defined as follows:

- Develop hardware and software for the control of a single and double pendulum
- Stabilize the single inverted pendulum
- Stabilize the double inverted pendulum
- Swinging up the single inverted pendulum

In order to have a well performing system to be used in control labs, there were several requirement for the system. These are itemized below:

- Ability to be controlled by MATLAB and Simulink
- Utilize the dsPIC as the controller
- Safe to be used by students
- Easy to setup and control
- Easy to convert from a single to a double and triple inverted pendulum

Based on those requirements and the project goals, sub-goals are defined and listed below:

- Design a double inverted pendulum with the ability to transform into single and triple inverted pendulum
- Purchase or manufacturing required parts
- Design the required circuitry and electrical hardware
- Purchase or develop the necessary electrical hardware
- Assemble the mechanical and electrical parts together to build the double inverted pendulum
- Interface dsPIC with brushless DC motor via PWM
- Condition encoder signals
- Derive dynamics equations of the pendulum
- Design a controller
- Implemente and debug software
- Caliberate and tune controller

B. Project motivation

A double inverted pendulum is a highly non-linear, multi-variable, high order, unstable system that consists of two pendulums in series connected to a cart that move freely along the horizontal axis. The inverted pendulum is a classic problem in dynamics and control theory and is widely used for testing many types of control algorithms. There are many practical problems for which the inverted pendulum is used as a representative model, such as the stabilization of humanoid or robot motion [1] and the launch phase of rockets.

C. Project background and literature

Many researchers have studied the pendulum system in depth and put forth various control methods. [2], [3]. These methods have important theoretical and practical significance. Sehgal et. al [4] has presented the model of a cart triple inverted pendulum system using Lagranges equation. They have linearized their model to design an LQR controller to maintain the triple inverted pendulum on a cart around its unstable equilibrium position using a single control input. C.J. Zhang et. al [5] proposed an optimization for defining L and Q

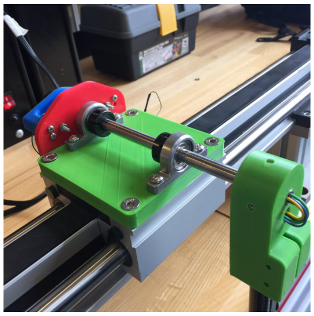

Fig. 1. First revolute joint between the carriage and the first link

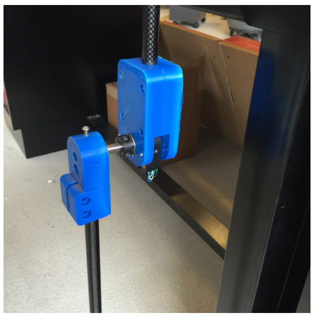

Fig. 2. Second revolute joint between the first and second links

matrices in the LQR method by genetic algorithm. Dynamics equations for double and triple inverted pendulums using Lagrangian and Newtonian approaches can be found in literature such as [6]. It is very common to write dynamics equations in state-space format to be easily handled by MATLAB. It should be mentioned that there are some companies such as Quanser which provide several types of inverted pendulums for educational use. They also have the ability to interface their hardware with MATLAB.

II. Device Discription

A. Mechanical design

Design of the double inverted pendulum was based on passing the aforementioned requirements. The linear rail and powertrain system were provided at the start of the project. Then joints and pendulum links were designed in SolidWorks and added into the rail and powertrain assembly. The first revolute joint was designed in a way that had two supporting bearings and enough space to mount an encoder and a slipring at one end of the shaft. The second revolute joint benefits from two bearings, one on each side of the encoder, to tolerate any exerted torque. The wires were hidden from sight by passing them through the center of the pendulums. All links, shafts and joint parts have holes inside for passing the wires through. All of the wire passageways correlate in a part to its mating parts. Hiding the wires makes the system look better and perform with less friction. Figures (1) and (2) illustrate the first and second joint of the pendulum.

The ease of 3D printing made it the main manufacturing process. All printing parts were designed in a way that they could be printed easily. It should be mentioned that some design of parts changed during the printing process to be easily printable.

Figure (3) represents the designed triple inverted pendumum in CAD software.

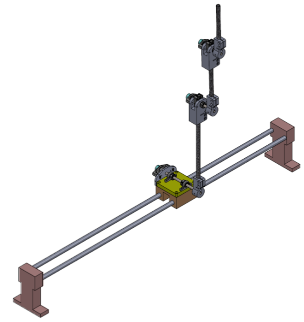

Fig. 3. CAD model of designed triple inverted pendulum

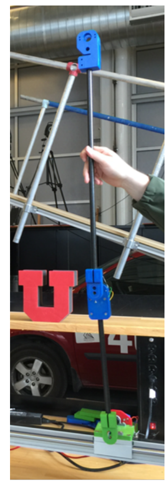

Fig. 4. Assembled double inverted pendulum

B. Manufacturing and assembly

To have light-weight parts, links are made of carbon fiber tubes and other parts are made of PLA or ABS. Once the printing of the parts was complete, the necessary holes were drilled and tapped. The bearings were pressed in and all other parts were assembled. Using the purchased spacing and alignment tools, each encoder was mounted. Wire routing involved a significant amount of soldering between the sliprings and encoder plugs. Assembled double inverted pendulum is shown in Fig. (4)

1) 3D printing: To do the manufacturing the parts much faster and specially have long holes in parts for passing wires, 3D printing selected as the main manufacturing process. PLA was the first material that used for printing but it was found that it very brittle specially for joint parts that were worked like a clamp around the carbun-fiber tubes. PLA part broke when just a little foce exerted on the part. ABS was the next choise and it had much better printing quality and flexiblity.

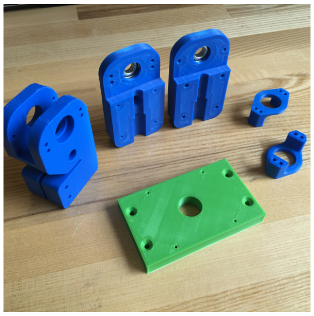

Fig. 5. 3D printed parts from ABS

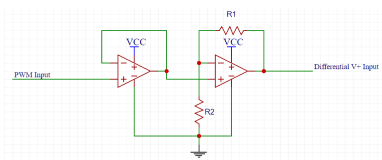

Fig. 6. Electrical schematic of PWM signal conditioning

The only problem was layer seperation in printing tall parts. Figure (5) shows the printed parts from ABS.

2) Tapping holes: Due to setting the printing parameters including layer thickness and shell thickness properly, tapping the holes in printed materials was very easy and there was a good grip between screw threads and hole threads which made the penduluml stiff and work well even after hitting the end of linear rail with high velocity.

C. Electrical hardware

The initial plan was to use a Keling brushless dc motor speed driver. This would have been significantly easier to implement due to its simple interface. This speed driver required a 0-5 V analog input for speed control and a single digital input for direction control. This driver was not working properly, so the Advanced Motion Controller AZB4 was chosen as its replacement. This platform was more difficult to integrate with the dsPIC as it required a 10V differential voltage input. The dsPIC was only capable of outputting a 3.3 V PWM output. The conversion from a 0-3.3V to a 10V differential input was accomplished through a mix of hardware and software. The first step was to take the 0-3.3V output and convert it to a 0-10V output. This was accomplished by running the PWM output first through a non-inverting buffer amplifier and then a non-inverting amplification with a gain of three. Fig. (6) shows the electrical schematic of PWM signal conditioning.

D. Electrical hardware

This provided the positive half of the needed range if the negative side of the differential input were tied to ground. To capture the negative side, it was decided to change the negative reference voltage to be +10 V through a digital output then provide the negative voltage by decreasing the duty cycle of the PWM output from 100% towards 0%. To convert the digital output from 0-3.3V to 0-10V, an identical

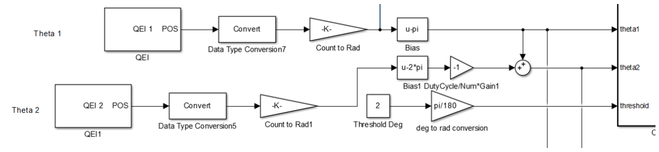

Fig. 7. Data aquisassion and inteprationg from encoders

buffer amplifier followed by a gain of three was used. The amplifiers utilized a 12V power supply. This allowed for a rail voltage higher than the desired output.

he project used two different types of encoders. For the pendulum angles discrete A and B channel type encoders were used. To determine the overall position of the carriage, an analog absolute encoder was used. The A B channel type of encoder required a 5V power supply. In order to minimize the number of different power supplies, a LM7805 linear voltage regulator stepped the 12V power supply down to 5V. The A and B channels of each encoder were connected to digital reprogrammable input pins of the dsPIC for use with the QEI module. The analog encoder required a 12V power supply and output a voltage proportional to the angle in a range of 0-5V. The dsPIC required an analog input of 0-3.3V; to convert between these two ranges, a voltage divider was used. Figure (8) represent the Sumulink blocks for data aquisassion from encoders.

E. Software

Programming of the dsPIC was initially done through the MPLABX IDE. This was suitable for the simple control of the single inverted pendulum. It was desired that the system be developed in a manner such that it could be incorporated into future controls class labs. Though it would be possible to require students to recreate or edit the existing C controller code, it was determined that it would be easier to allow the system to be represented through a MATLAB Simulink model then converted to C code using the Microchip Simulink Block set. The system could then be divided into subsystems and a high level of abstraction could be maintained for the lab user. All initial setup, calibration, and signal conditioning was completed in a subsystem block which from an onlookers point of view only requires a carriage force input and outputs all of the required states of the system. The user would simply need to close the loop and develop a controller around the subsystem.

To convert the PWM 0-10V input and 0—10V digital output to a 10V differential input, software was required to handle the correct conversion. The motor constant, gear ratio and motor controller gain were used to convert a desired carriage force to a duty cycle. This duty cycle could be either positive or negative depending on the desired direction of travel. If the value was negative, the negative voltage reference would remain low and the PWM duty cycle would be passed through to the motor driver. If the value was negative, the digital input would toggle high (10V) and the PWM duty cycle would be converted to have 100% equal the zero value and 0% to be the full -10V. For example, if the desired input were a -2.5V, or in other words a -25% duty cycle, the negative voltage reference would be toggled to 10V and the -25% would be converted to a 75% (7.5V) such that the differential output would be (Differential +) - (Differential -) or (7.5V)- (10V) = -2.5V.

The pendulum angles were tracked using the QEI module on the dsPIC. The output of the QEI module was a quadrature count that was converted to a pendulum angle through a gain block. For safety and controllability, it was desired that the stabilization controller would only become active once the pendulums came within 6 degrees of the zero angle position. This was done by routing the encoder angles through an embedded Matlab function to check angle positions at each sample and throw a controller activation flag if all encoders were within range. This flag was then used in another function to either pass through the control effort to the motor or disable it.

The absolute encoder was connected to the motor shaft. This meant that the encoder would revolve many times as the carriage moved from one extreme to the other on the rail. To track the cart position, each transition from 360 degrees to 0 or vice versa would need to be caught and accounted for. This tracking was done through an embedded MATLAB function and memory blocks. The last angle value was compared against the current value and used to determine if the revolutions counter should be indexed. Through a combination of this counter, the gear ratio and the relative motor angle, the overall carriage positon could be tracked.

Difficulty in using a LCD screen with the Simulink Model caused the need to utilize an alternate means of data output from the dsPIC. It was decided to add the ability to output any given variable, whether it be pendulum angle, control effort or voltage readings through a PWM output duty cycle. The duty cycle could be read through an oscilloscope and variable values could be known in real time.

F. Saftey

The inverted pendulum problem is inherently unstable and the linear rail was potentially over powered. These two considerations set safety at a high priority. Safety was handled through three different means.

-*Limit switches*: These switches were placed at the ends of travel. Once the carriage hit the travel limit of the linear rail, the limit switch would be tripped causing the motor to be disabled.

-*Control Limits*: As discussed in the software section, the control effort was only applied once the pendulum was near the zero angle position. If the pendulum were to go unstable and fall, the stabilization controller would turn off. This is essentially equivalent to disabling the motor.

-*Emergancy Stop*: Though not equipped with a push button E-Stop, an activation wire was put in place to engage the motor. In the event of a malfunction, the wire would be removed and the motor would disengage. An E-Stop bush button could easily replace the activation wire.

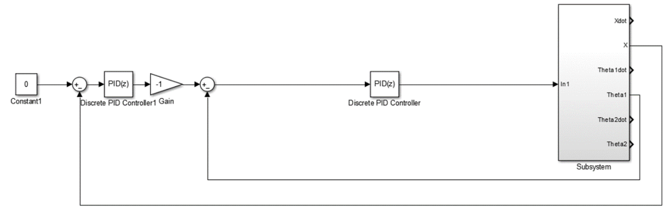

Fig. 8. PID control blocks in Simulink

III. Control and Performance

A. Control of single inverted pendulum

The first step was to control the stability of the single pendulum. Initially a long pendulum was put into use to test out different control methods. The long pendulum allowed for simple controllers to achieve stability, though with poor performance. The system was tested using a simple on-off bang bang controller. This control method did not achieve stability, but provided information that the controller was in negative feedback. Following this a proportional control was implemented. Through tuning, this controller provided stability of the long pendulum. Next, the pendulum was cut down to a 12 inch length. The shortening of the pendulum caused a full PID controller to be implemented in order to achieve acceptable performance. Simulink model of this controller is shown in Fig. (9) which includes a subsystem which is the Inverted pendulum and all blocks for communicating with dsPIC. The final goal was to control both stability and cart position. This was accomplished through cascade control. This control method functioned, but would have high overshoot on the position and was borderline oscillatory unstable.

B. Swinging up the single inverted pendulum

The swing up of the single pendulum was done open loop. Two pulses at 25% of the motor capacity were given at times corresponding to the pendulums natural frequency. This addition of inertia into the pendulum was sufficient to swing the pendulum into the upright position. Once the pendulum entered the controller threshold mentioned in the software and safety sections, the stabilization controller was activated and the pendulum stabilized.

C. Control of double inverted pendulum

With single pendulum stability, the next goal was to stabilize the double pendulum. To stabilize the inverted pendulum, it was found that a regulator controller was required. Because the system states are measureable, the system fulfills the observability requirement and state feedback could be used. For simplicity, a LQR controller was used. In order to find the optimal gain K, the system dynamics needed to be modeled.

1) Dynamic modeling: The Lagrangian approach was selected for modeling the dynamics of the system. The general Lagrangian equation of motion is as follows:

$$\frac{d}{dt}\left(\frac{\partial Tl}{\partial \dot{q^i}}\right) - \frac{\partial T}{\partial q^i} = Q_i \tag{1}$$

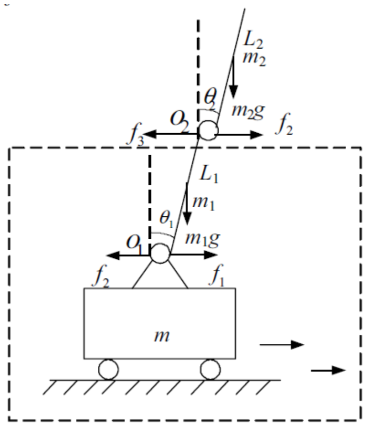

Fig. 9. Free-body diagram of a double inverted pendulum on a carriage [6]

where q_1, q_2,...,q_s are the generalized coordinates in the mechanical system under research; Q_1, Q_2 ,..., Q_s are the generalized coordinates on the mechanics system; q_1, q_2,...,q_s and t represent the generalized force; T is the kinetic energy which represented by the generalized coordinates q_1, q_2,...,q_s .

Free-body diagram for the double inverted pendulum on a carriage is represented in fig (1). Kinetic energy of carriage is

$$T_0 = \frac{1}{2}m_c\dot{x}^2 \tag{2}$$

and potential energy of carriage is obivous to be zero.

$$V_0 = 0 \tag{3}$$

For the first link of the pendulum, kinetic energy is

$$T_1 = \frac{1}{2}J_1\dot{\theta_1}^2 + \frac{1}{2}m_1\{[\frac{d}{dt}(x + l_1\sin\theta_1)]^2 + [\frac{d}{dt}(l_1\cos\theta_1)]^2\} \tag{4}$$

and potential energy is

$$V_1 = m_1gl_1\cos\theta_1 \tag{5}$$

Kinetic energy of the second link of the pendulum is

$$T_2 = \frac{1}{2}J_2\dot{\theta_2}^2 + \frac{1}{2}m_2\{[\frac{d}{dt}(x + l_1\sin\theta_1 + l_2\sin\theta_2)]^2 ... + [\frac{d}{dt}(l_1\cos\theta_1 + l_2\cos\theta_2)]^2\} \tag{6}$$

and its potential energy is derived from this equation:

$$V_2 = m_2g(l_1\cos\theta_1 + l_1\cos\theta_1) \tag{7}$$

So, kinetic energy of the whole system will be

$$\begin{aligned} T_{total} = T_0 + T_1 + T_2 = \frac{1}{2}m_c\dot{x}^2 + \frac{1}{2}J_1\dot{\theta_1}^2 \\ + \frac{1}{2}m_1\{[\frac{d}{dt}(x + l_1\sin\theta_1)]^2 + [\frac{d}{dt}(l_1\cos\theta_1)]^2\} \\ + \frac{1}{2}J_2\dot{\theta_2}^2 + \frac{1}{2}m_2[\frac{d}{dt}(x + l_1\sin\theta_1 + l_2\sin\theta_2)]^2 \\ + \frac{1}{2}m_2[\frac{d}{dt}(l_1\cos\theta_1 + l_2\cos\theta_2)]^2. \end{aligned} \tag{8}$$

The potential energy of system is as follow:

$$V_{total} = V_0 + V_1 + V_2 = m_1gl_1\cos\theta_1 + m_2g(l_1\cos\theta_1 + l_1\cos\theta_1) \tag{9}$$

Hence, the Lagrangian equation of motion for the double inverted pendulum is

$$L = L(x, \theta_1, \theta_2, \dot{x}, \dot{\theta}_1, \dot{\theta}_2) = T - V \tag{10}$$

The differential equations set of double inverted pendulum on carriage are as follow:

$$H_1(z)\ddot{z} = H_2(z, \dot{z})\dot{z} + h_3(z) + h_0u \tag{11}$$

where

$$z = (x, \theta_1, \theta_2)^T \tag{12}$$

$$h_0 = [1, 0, 0]^T \tag{13}$$

$$H_1(z) = \begin{bmatrix} a_0 & a_1\cos\theta_1 & a_2\cos theta_2 \\ a_1\cos\theta_1 & b_1 & a_2l_1\cos\theta_2 - \theta_1 \\ a_2\cos theta_2 & a_2l_1\cos\theta_2 - \theta_1 & b_2 \end{bmatrix} \tag{14}$$

$$H_2(z) = \begin{bmatrix} -f_0 & a_1\sin\theta_1.\dot{\theta}1 & a_2\sin theta_2.\dot{\theta_2} \\ 0 & -f_1 - f_2 & a_2l_1\sin\theta_2.\dot{\theta_2} \\ 0 & -a_2l_1\sin\theta_2 - \theta_1.\dot{\theta_1} + f_2 & -f_2 - f_3 \end{bmatrix} \tag{15}$$

$$h_3(z) = \begin{bmatrix} 0 & a_1g\sin\theta_1 & a_2g\sin\theta_2 \end{bmatrix}^T \tag{16}$$

where f_0 is the friction factor between carriage and linear rail, f_1 is friction factor in joint between carriage and the first link and and f_2 is the friction factor in joint between the first link and the secound link. Also,

$$a_0 = m_c + m_1 + m_2 \tag{17}$$

$$a_1 = m_1l_1 + m_2l_2 \tag{18}$$

$$a_2 = m_2l_2 \tag{19}$$

$$b_1 = J_1 + m_1l_1^2 + m_2l_2^2 \tag{20}$$

$$b_2 = J_2 + m_2l_2^2 \tag{21}$$

In the next step dynamic equation in () linerized about the uprite vertical position and transefered into state-space form as it is illustrated below:

$$X = \begin{bmatrix} x & \theta_1 & \theta_2 & \dot{x} & \dot{\theta_1} & \dot{\theta_2} \end{bmatrix}^T \tag{22}$$

$$\dot{X} = \begin{bmatrix} 0 & I_{4\times4} \\ E^{-1}H & E^{-1}H \end{bmatrix} X + \begin{bmatrix} 0 \\ E^{-1}h_0 \end{bmatrix} U = AX + BU \tag{23}$$

$$Y = CX \tag{24}$$

where

$$E = \begin{bmatrix} a_0 & a_1 & a_2 \\ a_1 & b_1 & a_2l_1 \\ a_2 & a_2l_1 & b_2 \end{bmatrix}^T \tag{25}$$

$$H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_1g & 0 \\ 0 & 0 & a_2g \end{bmatrix}^T \tag{26}$$

$$G = \begin{bmatrix} -f_0 & 0 & 0 \\ 0 & -f_1 - f_2 & f_2 \\ 0 & f_2 & -f_2 \end{bmatrix}^T \tag{27}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \tag{28}$$

based on the design of double inverted pendulum, design parameters are:

$$l_1 = 0.04064m, \qquad l_2 = 0.6878m$$

$$m_c = 1.5kg, \qquad m_1 = 0.4kg \qquad m_2 = 0.22kg$$

$$J_1 = 0.0407 \qquad J_2 = 0.359$$

$$f_0 = 0.5, \qquad f_1 = 0.2d \qquad f_2 = 0.2$$

2) LQR controller: After detemining state space modeling of the system, LQR method used to design a a regulator controller with good performance in stabilization of the double inverted pendulum. Since interface between MATLAB Simulink and dsPIC was ready, finding optimal K gain based on matrices A and B of the model and also Q and R matrices for LQR method would be easy. It was done by using a MATLAB command but still fining a proper Q and R matrices was a demanding task. For the start, R is selected to be 1 and Q is selected to be a diagonal matrix with Q_1,Q_2,Q_3 for penalizing position states and ,Q_4,Q_5,Q_6 for penalizing velocity states,in its diagonal values.

To have a better understanding of the effect of chaning each of those values in our system response, the system was successfuly simulated in MATLAB. However, due to lack of time, we could not reach to the optimal feedback gain K to stabilize the double inverted pendulum before the deadline. Reults for simulation of the system are illustrated in Fig. (11)

IV. Results and Achievements

Main achivements of this project are itemized here:

- Design of a triple inverted pendulum
- Manufacturing and assemble pendulums
- Interface dsPIC with advanced motion controller
- Interface dsPIC with encoders
- Stabilize single inverted pendulum
- Simulation the double inverted pendulum
- Interface dsPIC with MATLAB Simulink
- Swing up of the single inverted pendulum

V. Conclusion and Ongoing Works

The project was successful in that a double pendulum was designed, manufactured, and implemented. Despite having successful hardware and software a stable controller was not able to be achieved. Swing up and stabilization control was successful for the single inverted pendulum.

Future work should be done on both single and double inverted pendulum controller tuning. In addition, electrical hardware should be removed from the breadboard and implemented in a more robust printed circuit board. The power supply, dsPIC, rail and signal conditioning system should be hard mounted for ease of transport.

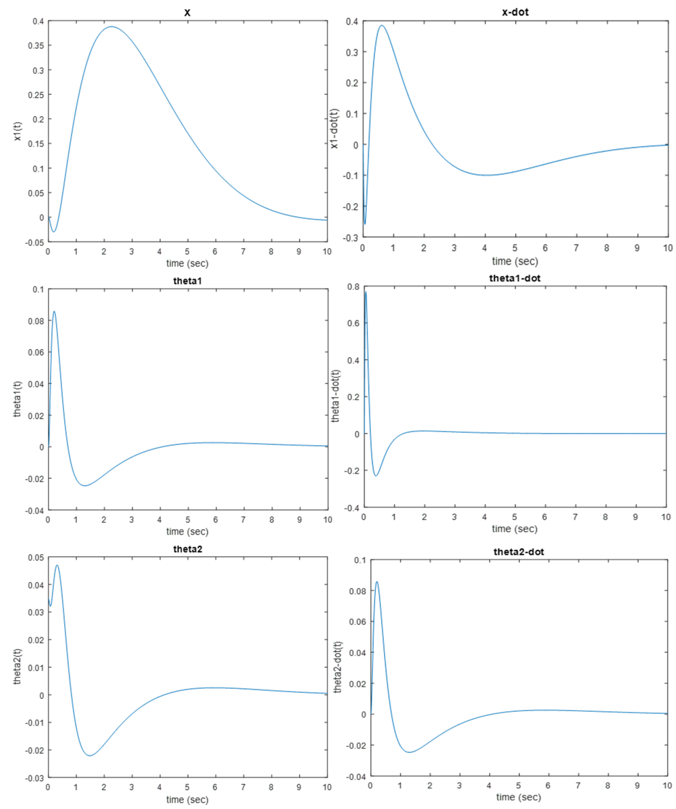

Fig. 10. Step-response of the double inverted pendulum in sumulation based on the design parameters

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